Roll Number		

SET B

INDIAN SCHOOL MUSCAT
FIRST PRELIMINARY EXAMINATION
MATHEMATICS

CLASS: XII   Sub.Code: 041   Time Allotted: 3 Hrs

10.01.2019   Max.Marks: 100

General Instructions:
(i) All questions are compulsory.
(ii) Questions in section A are very short answer type questions carrying 1 mark each.
(iii) Questions in section B are short- answer type questions carrying 2 marks each.
(iv) Questions in section C are long answer I type questions carrying 4 marks each.
(v) Questions in section D are long answer II type questions carrying 6 marks each.

SECTION- A (Questions 1 to 4 carry 1 mark each)

1. For what value of x, the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular?

2. A coin is tossed three times. Find $P(E|F)$ if, E = at most two tails; F = at least one tail.

OR

A card is drawn from a well- shuffled deck of 52 cards. If event E is that the card drawn is a club and event F is that the card drawn is an ace. Show that the two events are independent.

3. If $\frac{1}{2}\cos^{-1}\left(\frac{4}{5}\right) = \tan^{-1} x$, find the value of x.

4. The equation of a line is given by $\frac{2x-5}{4} = \frac{y+4}{3} = \frac{6-z}{6}$. Find the direction cosines of a line parallel to the above line.

SECTION- B (Questions 5 to 12 carry 2 marks each)

5. Find the value of λ, so that the following two lines are perpendicular:
$\frac{x-1}{4} = \frac{y+2}{3\lambda} = \frac{z-1}{5}$ and $\frac{x+1}{2\lambda} = \frac{y-1}{1} = \frac{z+2}{3}$.

6. Show that the vectors $\vec{a} = -2\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$, $\vec{b} = -2\hat{\imath} + 4\hat{\jmath} - 2\hat{k}$ and $\vec{c} = 4\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$ are coplanar.

7. Ten eggs are drawn successively with replacement from a lot containing 10% defective eggs. Find the probability that there is at least one defective egg.

8. Find the slopes of the tangent and the normal to the curve $y=\frac{(x-2)(x+1)}{(x+3)}$ at the points where it cuts the X-axis.

OR

Prove that the function $y=\left[\frac{4\,sinx}{2+\cos x}-x\right]$ is an increasing function of $x$in $\left(0,\frac{\pi}{2}\right)$.

9. Find the matrix X so that $X\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$

10. Find the mean and variance of the number of tails in three tosses of a fair coin.

11. Find the equation of the plane that contains the point (-1, 3, 2) and perpendicular to each of the planes $x+2y-3z=5$and $3x+3y-z=0$.

OR

Find the equation of the plane passing through the intersection of the planes $3x-y+2z-4=0$ and $x+y+z-2=0$ and through the point (2, 2, 1).

12. Form the differential equation of the family of circles touching the X- axis at origin.

<u>SECTION- C (Questions 13 to 23 carry 4 marks each)</u>

13. Evaluate: $\int_0^{\pi/2}\left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx$

OR

Evaluate using properties of definite integrals : $\int_0^{\pi}\frac{x}{a^2cos^2x+b^2sin^2x}dx$

14. Evaluate $\int_1^3(3x^2-5)dx$ by the method of limit of sum.

15. Using properties of determinants, prove that: $\begin{vmatrix}a+bx^2 & c+dx^2 & p+qx^2\\ ax^2+b & cx^2+d & px^2+q\\ u & v & w\end{vmatrix}=$

$(x^4-1)\begin{vmatrix}b & d & q\\ a & c & p\\ u & v & w\end{vmatrix}$

OR

Using properties of determinants, solve for x: $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0.$

16. If $\vec{a} = \hat{\imath} + 4\hat{\jmath} + 2\hat{k},\ \vec{b} = 3\hat{\imath} - 2\hat{\jmath} + 7\hat{k}$ and $\vec{c} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d} = 15$.

17. Evaluate : $\int \frac{2x^2+5x+7}{(x-2)(x-3)^2} dx$

18. If $f(x) = \begin{cases} ax+b & if \quad x > 2 \\ 7x-4 & if \quad x = 2 \\ 3ax-2b & if \quad x < 2 \end{cases}$ is continuous at $x = 2$, find the values of a and b.

19. Solve the differential equation: $cos^2 x \left(\frac{dy}{dx}\right) + y = tanx$.

OR

Solve the differential equation: $ye^y\, dx = (y^3 + 2xe^y)\, dy$, given that $y(0) = 1$.

20. Find the particular solution of the differential equation given that $y = 1$ when $x = 1$: $(3xy + y^2)dx + (x^2 + xy)dy = 0$.

21. In a group of 400 people, 160 are smokers and non-vegetarians, 100 are smokers and vegetarians and the remaining are non- smokers and vegetarians. The probabilities of getting a particular chest disease are 35%, 20% and 10% respectively. A person is chosen from the group at random and is found to be suffering from the chest disease. What is the probability that the selected person is a smoker and non- vegetarian?

22. Differentiate $y = x^{tanx} + (tanx)^x$ and find $\frac{dy}{dx}$.

23. Show that: $tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right] = \frac{\pi}{4} + \frac{1}{2}cos^{-1}x^2$

SECTION- D (Questions 24 to 29 carry 6 marks each)

24. Show that the semi- vertical angle of a cone of maximum volume and given slant height is $\tan^{-1}(\sqrt{2})$.

25. Using integration, find the area of the region bounded by the line $x - y + 2 = 0$ and the curve $x^2 = y$.

OR

Using integration, find the area of the following region: $\{(x, y): |x-1| \le y \le \sqrt{5-x^2}\}$

26. Show that the lines $\frac{x-1}{2} = \frac{y-1}{3} = \frac{z-3}{-5}$ and $\frac{x-3}{1} = \frac{y-4}{-3} = \frac{z+2}{2}$ intersect. Also find the point of intersection.

27. If $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 3 & -1 \\ -2 & 1 & 1 \end{bmatrix}$, find A^{-1} and use A^{-1} to solve the following system of equations:

$$2x + y + 3z = 9, \qquad x + 3y - z = 2, -2x + y + z = 7.$$

OR

Using elementary transformation, find the inverse of the matrix: $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

28. A manufacturer makes two types of products A and B. Both products are processed on two different machines. The available capacity of first machine is 12 hours and that of second machine is 9 hours per day. Each unit of the product A requires 3 hours on both machines and each unit of product B requires 2 hours on first machine and 1 hour on second machine. Each unit of product A is sold at a profit of Rs.7 and each unit of product B at a profit of Rs.4. Find the production level per day for maximum profit graphically. Also find the maximum profit.

29. A binary operation $*$ is defined on the set X = R-{-1} by $x * y = x + y + xy,\ \ x, y \epsilon$ X. Check whether $*$ is commutative and associative. Find the identity element and also find the inverse of each element of X.

OR

Let A = Q $\times$Q where Q is the set of rational numbers and $*$ be the binary operation on A defined by $(a, b) * (c, d) = (ac, b + ad)$ for $(a, b), (c, d) \epsilon\ A$. Then find:
(i) the identity element of $*$ in A.
(ii) invertible element of (a, b) and hence write the inverse of elements (5, 3) and $\left(\frac{1}{2}, 4\right)$.

End of the Question Paper